Introduction

This document has been adapted from the guidance produced by the North Yorkshire County Council maths team to support schools in the teaching of written calculations in line with the new National Curriculum Programmes of Study for Mathematics 2014. It draws upon a range of existing freely and publicly available publications.

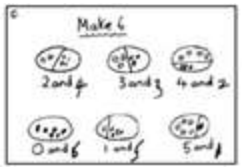
It is designed to present agreed, consistent and systematic approaches the school will take in the teaching of written calculations.

Staff may find that resources from www.ncetm.org.uk are helpful, particularly the suite of videos which include examples of some of the approaches suggested in this documentations in practice in the classroom. These can be found at https://www.ncetm.org.uk/resources/40529

Staff looking for support in exploring the rationale behind teaching children to use written calculations and the necessary skills, knowledge and understanding that will underpin this work may find the document 'Teaching Written Calculations: Guidance for teachers at Key Stages One and Two' helpful. This and many other documents produced through the National and Primary strategies can be found in the e-library of the STEM centre at http://www.nationalstemcentre.org.uk/elibrary/

Mental and written calculation methods should be taught alongside each other throughout the entirety of this progression. When teaching children to calculate emphasis should be placed on choosing and using the method that is most efficient. If a child can complete a calculation mentally or with jottings, they should not be expected to complete a written algorithm. Whilst no longer part of the statutory curriculum, children should also be taught when and how to use a calculator appropriately.

ADDITION

Guidance	Examples	
Stage 1: Recording and developing mental pictures • Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.	**Stage 1**  **One and one, two more**  **makes one, two three."** There are 3 people on the bus. Another person gets on. How many now? 	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process. Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.

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Stage 2: Progression in the use of a number line

• To help children develop a sound understanding of numbers and to be able to use them confidently in calculation, there needs to progression in their use of number tracks and number lines

The labelled number line

• Children begin to use numbered lines to support their calculations counting on in ones.

• They select the biggest number first i.e. 8 and count on the smaller number in ones.

Stage 2

Children should experience a range of representations of number lines, such as the progression listed below.

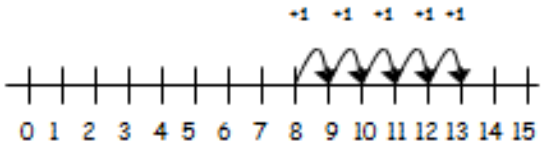
Number track

1	2	3	4	5	6	7	8	9	10

Number line, all numbers labelled

- Number line, 5s and 10s labelled
- Number line, 10s labelled
- Number lines, marked but unlabelled

8 + 5 = 13



Additional 'number lines' - The bead string and hundred square

A hundred square is an efficient visual resource to support adding on in ones and tens and is an extension to the number track that children have experienced previously.

8 + 2 = 10

Different orientations of the 100 square help children transfer their skills and understanding between similar representations.

Along with the number line, bead strings can be used to illustrate addition. Eight beads are

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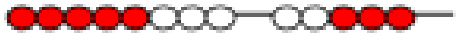
		counted out, then the two beads. Children count on from eight as they add the two beads e.g. starting at 8 they count 9 then 10 as they move the beads. Eight beads are counted out, then the five. Children count on from eight as they add the five e.g. starting at 8 they count 9, 10, 11, 12, 13.  8 + 5 = 13 

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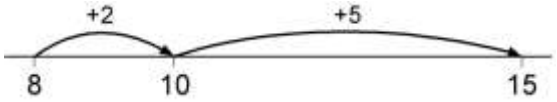
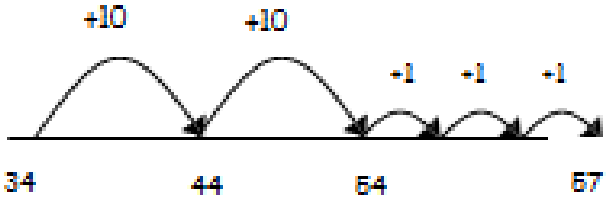
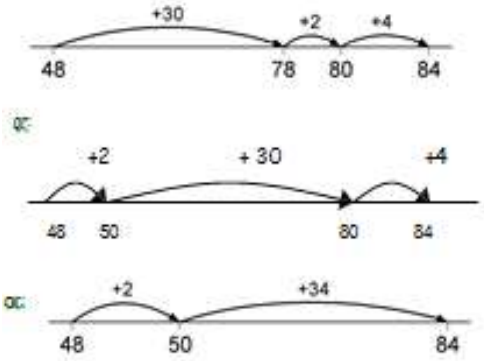
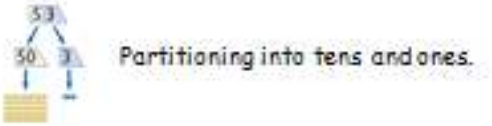
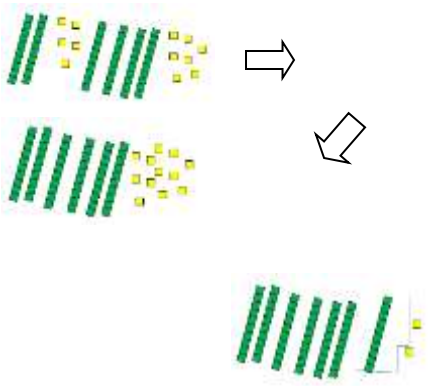
Stage 3: The empty number line as a representation of a mental strategy **NB It is important to note that the empty number line is intended to be a representation of a mental method, not a written algorithm (method). Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.** • The mental methods that lead to column addition generally involve partitioning. • <u>Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.</u> • The empty number line helps to record the steps on the way to calculating the total.	Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 8 + 7 = 15 Seven is partitioned into 2 and 5; 2 creating a number bond to 10 with the 8 and then the 5 is added to the 10.  First counting on in tens and ones. 34 + 23 = 57  This develops in efficiency, alongside children's confidence with place value.	Counting on in multiples of 10. 48 + 36 = 84  These examples show how children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating with. **This reinforces that this is a visual representation of a mental method and not a written alogrithm.**

Mental and written calculation methods should be taught alongside each other throughout the entirety of this progression. When teaching children to calculate emphasis should be placed on choosing and using the method that is most efficient. If a child can complete a calculation mentally or with jottings, they should not be expected to complete a written algorithm. Whilst no longer part of the statutory curriculum, children should also be taught when and how to use a calculator appropriately.

Stage 4: Partitioning into tens and ones to lead to a formal written method • The next stage is to record mental methods using partitioning into tens and ones separately.  Partitioning into tens and ones. • Add the tens and then the ones to form partial sums and then add these partial sums. • Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods. • This method can be extended for TU + HTU and HTU + HTU and beyond; as well as cater for the addition of decimal numbers.	Stage 3 Children should use a range of practical apparatus (place value cards, straws, Dienes apparatus, place value counters) to complete TU + TU. They partition the number into tens and ones before adding the numbers together, finding the total. There should be progression through this selection of apparatus. Once using abstract representations teachers will start with straws, bundled into 10s and singularly. Children see 10 straws making one bundle and can be involved in bundling and unbundling. This then progresses to the use of Dienes (or similar) where 10s are clearly ten ones but cannot be separated in the same way. Once children are able to use these with understanding, they will progress to the use of place value cards and place value counters which are a further abstraction of the concept of number. Money should also be used (1ps, 10ps and £1) as place value equipment to help children develop their understanding of a range of representations. Progress through these manipulatives should be guided by understanding not age or year group.	15 + 47  Children may make these jottings to support their calculation. 47 + 76 40 + 70 = 110 **or** 7 + 6 = 13 7 + 6 =13 40 + 70 = 110 110+ 13 = 123 110 + 13 = 123 **or** 47 + 70 = 117 117 + 6 = 123

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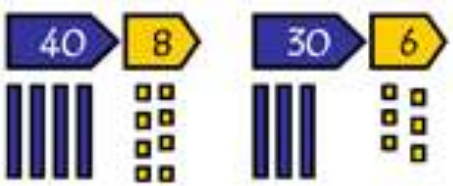
	48 + 36  40 + 30 = 70 8 + 6 = 14 70 + 14 = 84 Cuisenaire can also be used to support this step, especially when crossing the tens barrier with ones. When this occurs, children should use the term 'exchange' to describe converting ten ones into one ten.	
Stage 5 – Using Dienes/place value counters alongside columnar written method • To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout. • Children first experience the practical version of column addition and when confident in explaining this, including	**It may be appropriate to teach children the process with numbers that they would be expected to calculate mentally or with jottings. This is to aid with the practicalities of the use of such equipment. However this should be the exception rather than the rule so children see a clear purpose for learning a new method for calculating.**	

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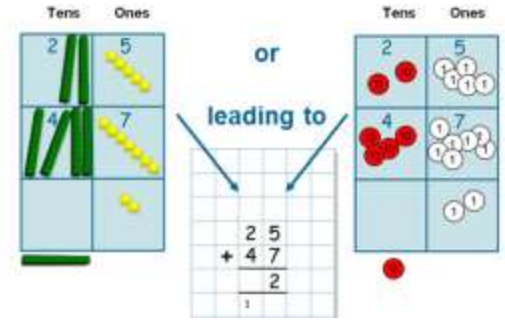
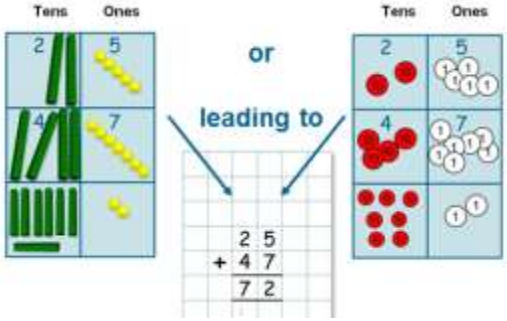
exchanging when crossing the tens barrier with ones, they record the written method alongside. • Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations. • Children may learn more from experiencing the inefficiency of not starting with column with least significant value rather than being 'told' where to start.	In this example 25 + 47 =   Whilst these images show the total existing alongside the two numbers being added, it may be more representative to 'drag' the manipulatives down to the totals box, leaving	Represented in place value columns and rows. Starting adding with the 'least significant digit' When the tens barriers is crossed in the 'ones' exchange then takes place. Because of the exchange we can know see that this ten belongs in the tens column and is carried there to be included in the total of that column. The tens are then added together 20 + 40 + 10 = 70, recorded as 7 in the tens column. This method aligns with the approaches used in some intervention programmes and involves less movement of equipment however does not match as

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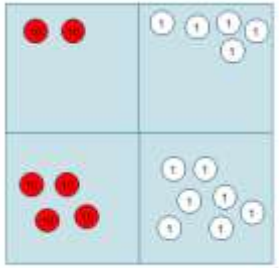
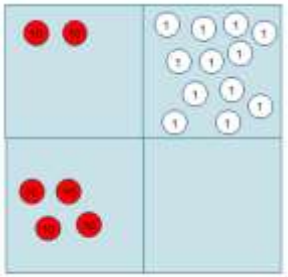
	the written numbers as a reminder of what was originally there. Another way of representing this is like this  25 +47 Where the bottom value is combined with the top value  25 +47	closely to the columnar abstract representation as the suggestion above.

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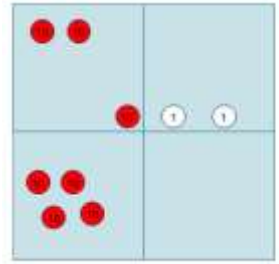
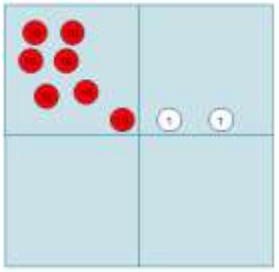
	 25 +47 2  25 +47 72	

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Stage 5: Compact column method • In this method, recording is reduced further. Carried digits are recorded, using the words 'carry ten' or 'carry one hundred' etc., according to the value of the digit. • Children will be taught to write the carrying digit underneath the line, in the column to which it will be added. A consistent approach to the location of carried digits and encouraging children to be consistent with their placement should reduce error rates. However as long as children understand the carried digit, the location of the recording is unimportant and this should be made clear to children. • Later the method is extended when adding more complex combinations such as three two-digit numbers, two three-digit numbers, and problems involving several numbers of different sizes.	**Stage 5** 258 + 87 = 345 (1 1) 366 + 458 = 824 (1 1) **Column addition remains efficient when used with larger whole numbers and once learned, is quick and reliable.** 3674 + 2507 = 6181 (1 1)	 Method agreed at staff meeting 22.4.15 etc

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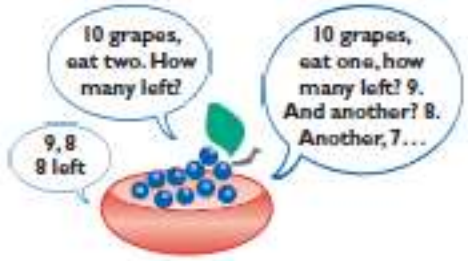
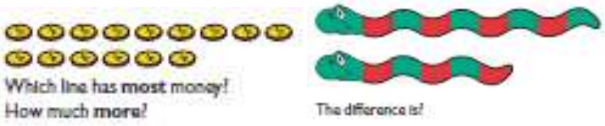
SUBTRACTION

Guidance	Examples	
Stage 1: Recording and developing mental pictures Children are encouraged to develop a mental picture of the calculation in their heads. They experience practical activities using a variety of equipment and develop ways to record their findings including models and pictures. • The 'difference between' is introduced through practical situations and images.	**Stage 1**  There are four children in the home corner. One leaves. How many are left?  	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process. Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.

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Stage 2: Progression in the use of a number line

• Finding out how many items are left after some have been 'taken away' is initially supported with a number track followed by labelled, unlabelled and finally empty number lines, as with addition.

The labelled number line

• The labelled number line, linked with previous learning experiences, is used to support calculations where the result is less objects (i.e. taking away) by counting back.

Difference between

• The number line should also be used to make comparisons between numbers, to show that 6 –

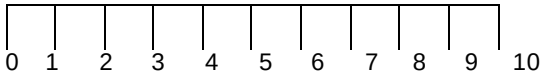
Stage 2

Children should experience a range of representations of number lines, such as the progression listed below.

Number track

1	2	3	4	5	6	7	8	9	10

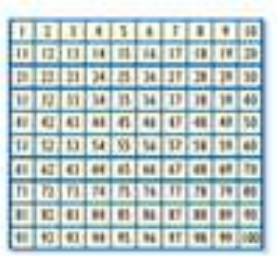
Number line, all numbers labelled



- Number line, 5s and 10s labelled
- Number line, 10s labelled
- Number lines, marked but unlabelled

Additional 'number lines' - The bead string and hundred square

A hundred square is an efficient visual resource to support counting on and back in ones and tens and is an extension of the number track which children have experienced previously



Different orientations of the 100 square help children transfer their skills and understanding between similar representations.

- Bead strings can be used to illustrate subtraction. 6 beads are counted and then the 2 beads taken away to leave 4.

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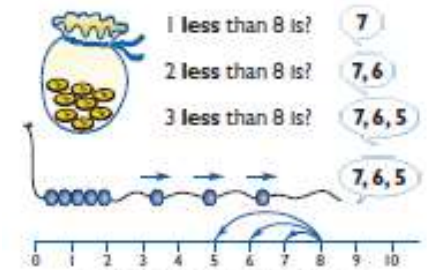
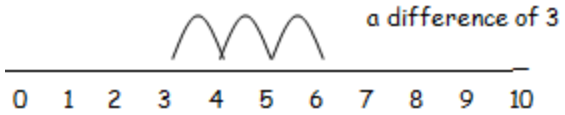
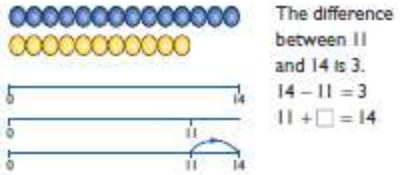
3 means the '**difference in value** between 6 and 3' or the 'difference between3 and 6' and how many jumps they are apart.	  	
Stage 3: The empty number line as a representation of a mental strategy **NB It is important to note that the empty number line**	Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10. 15 – 7 = 8	The steps may be recorded in a different order:

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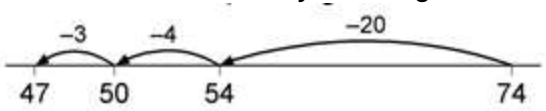
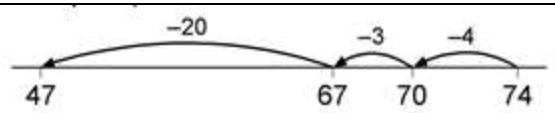
is intended to be a representation of a mental method, not a written algorithm (method). Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next. **Finding an answer by COUNTING BACK** • **Counting back** is a useful strategy when the context of the problem results in there being less e.g. Bill has 15 sweets and gives 7 to his friend Jack, how many does he have left? As in addition, children need to be able to partition numbers e.g. the 7 is partitioned into 5 and 2 to enable counting back to 10. • The empty number line helps to record or	The seven is partitioned into 5 (to allow count back to 10) and two.  74 – 27 = 47 worked by counting back: 	 or combined  These examples show how children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating with. **This reinforces that this is a visual representation of a mental method and not a written alogrithm.**

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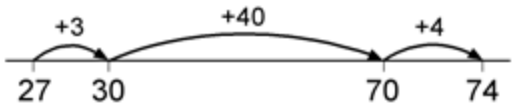
explain the steps in mental subtraction. • A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is a useful way of modelling processes such as bridging through a multiple of ten.		
Using an empty number line **Finding an answer by COUNTING ON** • The steps can also be recorded by **counting on** from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shopkeeper's method). This is a useful method when the context asks for comparisons e.g. how much longer, how much smaller; for example: Jill has knitted 27cm of her scarf, Alex has knitted 74cm.	74 - 27 =  The 'jumps' should be added, either mentally or with jottings according to confidence, beginning with the largest number e.g. 40 + 4 + 3. or	

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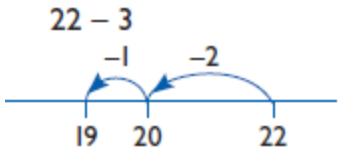
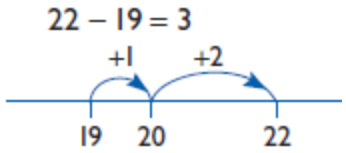
How much longer is Alex's scarf? • After practice of both, examples like this will illustrate how children might choose when it is appropriate to count on or back. This also helps to reinforce addition and subtraction as inverses and the links between known number facts.	  	

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Stage 4: Practical equipment using exchange to 'take away' • Children use practical apparatus to take away the smaller number from the larger. This should be used to model exchanging as in the example. • Children's place value knowledge should be good enough to understand that the change still represents the original starting number and is just a different way of partitioning it.	**It may be appropriate to teach children the process with numbers that they would be expected to calculate mentally or with jottings. This is to aid with the practicalities of the use of such equipment. However this should be the exception rather than the rule so children see a clear purpose for learning a new method for calculating.** There should be progression through this selection of apparatus. Once using abstract representations teachers will start with straws, bundled into 10s and singularly. Children see 10 straws making one bundle and can be involved in bundling and unbundling. This then progresses to the use of Dienes (or similar) where 10s are clearly ten ones but cannot be separated in the same way. Once children are able to use these with understanding, they will progress to the use of place value cards and place value counters which are a further abstraction of the concept of number. Money should also be used (1ps, 10ps and £1) as place value equipment to help children develop their understanding of a range of representations.	

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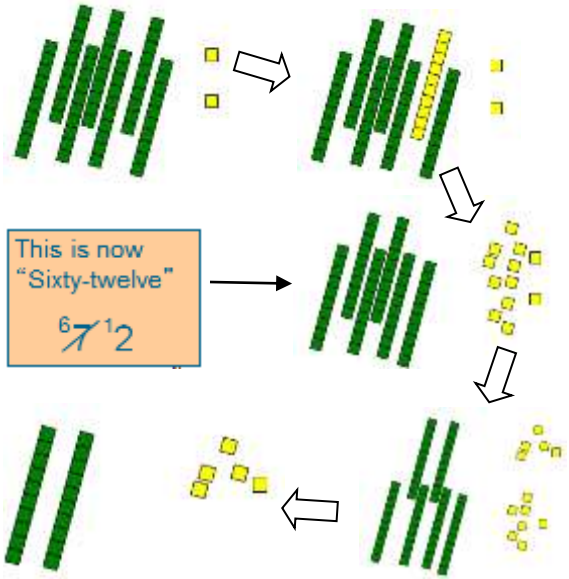
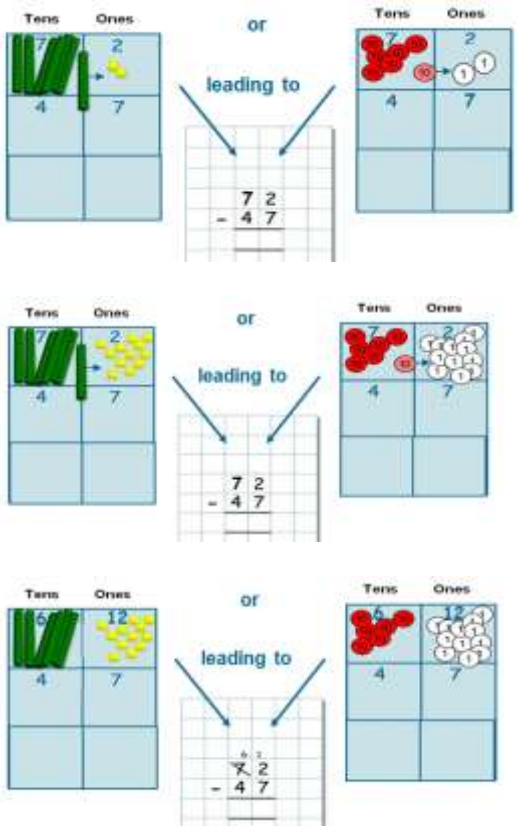
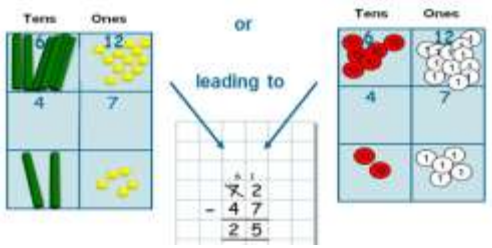
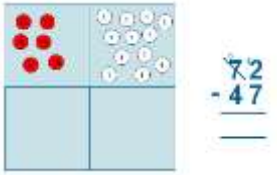
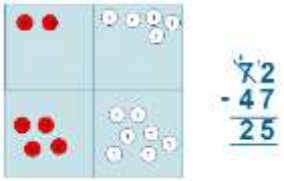
	Progress through these manipulatives should be guided by understanding not age or year group. 72 – 47  72 – 47 = 25	This stage should also be represented using place value counters, going through the same process as the Dienes example. Because of the cumbersome nature of 'exchanges' in this form, examples that children are expected to do with the practical equipment should be limited to HTU – HTU with one exchange in each calculation

Mental and written calculation methods should be taught alongside each other throughout the entirety of this progression. When teaching children to calculate emphasis should be placed on choosing and using the method that is most efficient. If a child can complete a calculation mentally or with jottings, they should not be expected to complete a written algorithm. Whilst no longer part of the statutory curriculum, children should also be taught when and how to use a calculator appropriately.

Stage 5: Making the link between the practical and columnar subtraction • To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout. • Children first experience the practical version of column subtraction and when confident in explaining this, including exchanging when 'not having enough to subtract from', they record the written method alongside. • Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations.	72 - 47 	

Mental and written calculation methods should be taught alongside each other throughout the entirety of this progression. When teaching children to calculate emphasis should be placed on choosing and using the method that is most efficient. If a child can complete a calculation mentally or with jottings, they should not be expected to complete a written algorithm. Whilst no longer part of the statutory curriculum, children should also be taught when and how to use a calculator appropriately.

• Children may learn more from experiencing the inefficiency of not starting with column with least significant value than being 'told' where to start.	 Another way of representing this is;  Where the subtracted amount is removed  And the answer is represented by what is remaining in the top row	Whilst the images here show the total existing alongside the original number, it is suggested that the 47 would be 'removed' from the original set, before 'dragging' what is left down to the totals box. This would more closely represent the written algorithm. This method aligns with the approaches used in some interventions and involves less movement of equipment however does not match as closely to the columnar abstract representation as the suggestion above.

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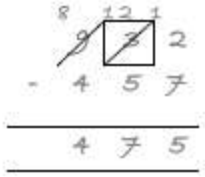
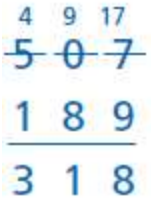
Stage 6: Compact method • Finally children complete the compact columnar subtraction as the most efficient form. • Once children are confident with HTU – HTU, this should be extended to four digit subtract four digit calculations.	563 – 246 = 317 5 1 5 6 3 2 4 6 3 1 7 932 - 457 becomes  Answer: 475	 Children may find it more helpful to present their exchanges like this to keep the numbers clear.  Method agreed at staff meeting 22.4.15 However if individual children use s33lightly different locations for their representations of exchange but understand and can explain what they are doing and why, the placement is not important.

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MULTIPLICATION

Guidance	Examples	
Stage 1: Recording and developing mental images • Children will experience equal groups of objects. They will count in 2s and 10s and begin to count in 5s. • They will experience practical calculation opportunities involving equal sets or groups using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. • They develop ways of recording calculations using pictures, etc. • They will see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning answering questions such as; 'How many eggs would we need to fill the egg box? How do you know?' • Children will use repeated addition	**Stage 1**  2 + 2 + 2 + 2 + 2 = 10  5 + 5 + 5 + 5 + 5 + 5 = 30 5 × 6 = 30  2 groups of 3 are 6 (3 + 3) 3 groups of 2 are 6 (2 + 2 + 2)	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process. Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.

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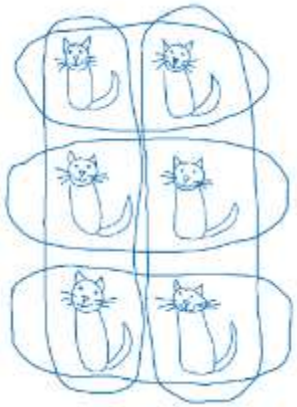
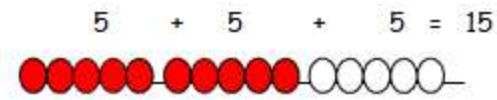
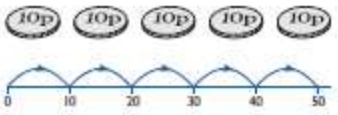
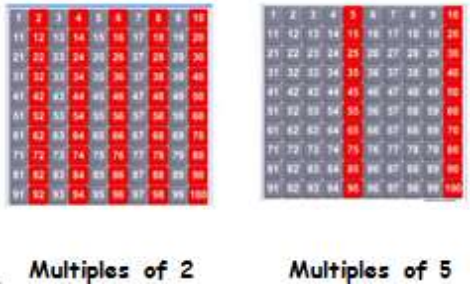
to carry out multiplication supported by the use of counters/cubes.	 4 lots of 3 are 12 3 lots of 4 are 12  Children should use pictorial representations and may use rings to show e.g. 3 groups of 2 and 2 groups of 3 introducing the commutative law of multiplication	

Mental and written calculation methods should be taught alongside each other throughout the entirety of this progression. When teaching children to calculate emphasis should be placed on choosing and using the method that is most efficient. If a child can complete a calculation mentally or with jottings, they should not be expected to complete a written algorithm. Whilst no longer part of the statutory curriculum, children should also be taught when and how to use a calculator appropriately.

Stage 2: The bead string, number line and hundred square • Children continue to use repeated addition to carry out multiplication tasks and represent their counting on a bead string or a number line. • On a bead string, children count out three lots of 5 then count the beads altogether. • On a number line. Children count on in groups of 5. • These models illustrate how multiplication relates to repeated addition.	**Stage 2** 3 lots of 5   10p + 10p + 10p + 10p + 10p = 50p 10p × 5 = 50p 5 hops of 10	Children begin pattern work on a 100 square to help them begin to recognise multiples and rules of divisibility.  Children regularly sing songs, chant and play games to reinforce times tables facts and their associated patterns.

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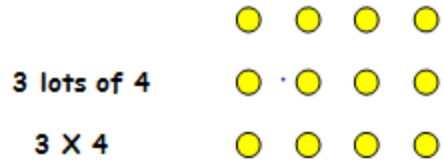
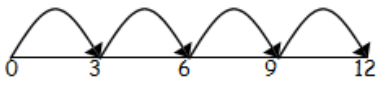
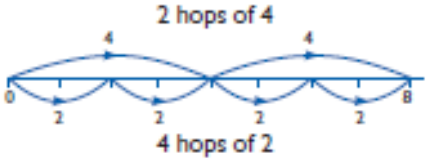
Stage 3: Arrays It is important to be able to visualise multiplication as a rectangular array. This helps children develop their understanding of the commutative law i.e. 3 x 4 = 4 x 3 The rectangular array allows the total to be found by repeated addition and the link can be made to the 'x' sign and associated vocabulary of 'lots of' 'groups of' etc.	**Stage 3**  3 lots of 4 3 X 4 4 lots of 3 4 x 3  4 lots of 3 = 12  3 lots of 4 =12	The relationship between the array and the number line showing both repeated additions should be demonstrated alongside each other  For more direct comparison, this could then be demonstrated on a single number line as appropriate.

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<table>
<tr>
<td>Related calculations and estimates
To utilize further methods, children need to
a) know their multiplication facts up to 10 x 10
b) be able to identify and use related calculations and place value effectively
E.g. for 47 X 6 they must be able to calculate 40 X 6. They need to recognise the ‘root’ calculation
4 x 6 = 24
and understand that as 40 is ten times greater than 4 the product will also be ten times greater.
40 x 7 = 240
Before carrying out calculations children are encouraged to estimate their answer using rounding. In the first instance, this might be;
- in the case of a 2 digit number to the nearest 10 or a 3 digit number to the nearest 100.
They compare their answer with the estimate to check for reasonableness.</td>
<td>47 x 6
Estimate 47 x 6 is approximately 50 x 6 = 300
<table><tr><td>X</td><td>40</td><td>7</td></tr><tr><td>6</td><td>240</td><td>42</td></tr></table>240 + 42 = 282

Check against estimation – 282 is less than 300 but as 47 was rounded UP to 50 the answer seems reasonable.</td>
<td></td>
</tr>
</table>

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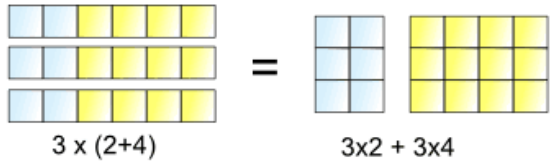
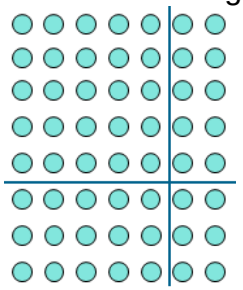
Stage 4: The Grid Method	**Stage 4**	**NB examples such as this should only be used as a model to represent the transition between arrays and the grid method, not to calculate with. Children should be expected to complete calculations of this nature mentally, or with jottings if needed.**
• This is the first exposure to the distributive law of multiplication and children should be given plenty of opportunity to explore this • Children will partition arrays in a variety of helpful ways which are not necessarily the ways in which they will eventually partition them to be in line with formal written methods • The link between arrays and the grid method should be made clear to children by the use of place value apparatus such as place value counters and Dienes. • The TU number is partitioned e.g. 13 becomes 10 and 3 and each part of the number is them multiplied by 4.	 The outcome of 3 x 6 will be the same as 3 x 6 partitioned (in this example into 2 + 4) Use larger preconstructed arrays to look at ways these can be partitioned to use already know number facts e.g 7 x 8  4 x 13	Knowing 5 and 2 x tables and being able to add, I can partition this array to use these facts to work. 5 x 5 = 25, 5 x 3 = 15, 5 x 2 = 10 2 x 3 = 6 7 x 8 = 25 + 15 + 10 + 6 = 56

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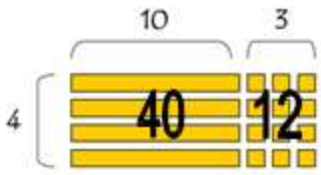
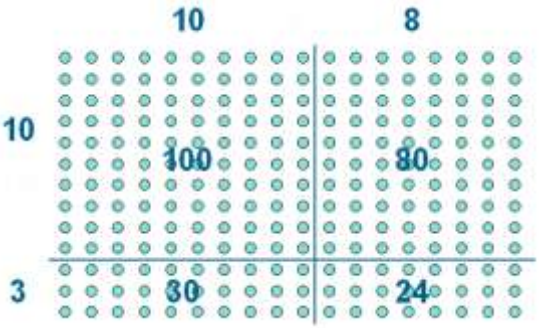


Children should be shown how this model shows 4 x 13 but the calculation steps are 'made easier' by partitioning the 13 into 10 and 3. The use of Dienes emphasises the distributive law.

This then becomes

x	**10**	**3**
4	40	12

40 + 12 = 52



Adding the rows or adding the columns

This should be decided by the child depending on the numbers that are produced through the calculation.

53 x 16
Estimate 50 x 20 = 1000

x	10	6
50	500	300
3	30	18

Adding the rows is the most efficient calculation:
500 + 300 = 800
30 + 18 = 48
So 800 + 48 = 848
848 is quite a distance from 1000 but rounding 53 and 16 to the nearest ten was also a significant difference.
The answer is reasonable.

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<table>
<tr>
<td>Two-digit by two-digit products using the grid method (TU x TU)
Children first make an estimate by rounding each number to the nearest ten.
Having calculated the sections of the grid, children will decide whether to add the rows or columns first as they become more confident with recognising efficient calculations.
They will choose jottings, informal or formal written methods depending upon which is most appropriate.
Children should be expected to complete this for TU X TU but not for larger numbers.</td>
<td>Using pre constructed arrays, children look for ways to split them up using number facts that they are familiar with. Over time this leads to children partitioning two digit numbers into tens and ones, making the link to grid multiplication which is a pre cursor to short and long multiplication.

Children move to the grid method without arrays once they can confidently explain the relationship between the two, even when the array is no longer visible.</td>
<td>38 x 17
Estimate 40 x 20 = 800
<table><tr><td>x</td><td>10</td><td>7</td></tr><tr><td>30</td><td>300</td><td>210</td></tr><tr><td>8</td><td>80</td><td>56</td></tr></table>Adding the columns:
300 + 80 = 380
210 + 56 = 266
380 + 200 = 580 + 60 = 640 + 6 = 646
As both 38 and 17 were round up, the higher estimation of 800 is expected and makes this answer reasonable.</td>
</tr>
</table>

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Stage 5: Expanded short multiplication • The first step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above. • Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.	**Stage 5** Multiply the units first which enables them to move towards the compact method e.g. 30 + 8 X 7 56 7 x 8 210 7 x 30 266	

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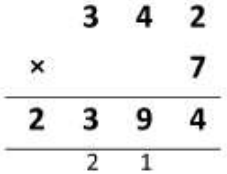
Stage 6: Short multiplication for up to TU x 12

- The recording is reduced further, with the carried digits recorded either below the line or at the top of the next column.
- This method is appropriate for multiplying two and three digit numbers by numbers up to 12, which relies on children have recall of their times table facts up to 12.

342 × 7 becomes



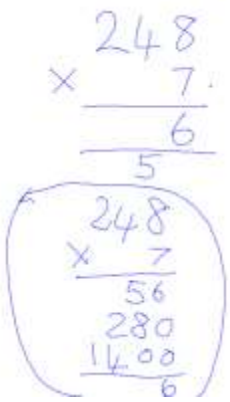
Answer: 2394

This example shows the carried digits below the line

124 × 12 becomes

Answer: 1 488

This example shows the carried digits at the top of the next column



Method agreed at staff meeting 22.4.15

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Expanded long multiplication (TU x TU)

- To ensure understanding of this method, it is important to make direct links to the grid method and may be helpful in the first instance to do both methods side by side to allow children to see the relationship
- There should be an emphasis on making sure that each part of each number is multiplies by each part of the other number

To use the example from earlier in this document

53 x 16

x	10	6
50	500	300
3	30	18

53
x 16
500 (50 x 10)
300 (50 x 6)
30 (3 x 10)
+ 18 (3 x 6)
848

Staring with the most significant digits makes a more direct link to the grid method.

Children should be moved towards starting with the column of smallest value as soon as their understanding of the relationship between the methods allows, to move towards long multiplication.

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<table>
<tr>
<td></td>
<td>
53
x 16
18 (6 x 3)
300 (6 x 50)
30 (10 x 3)
+500 (10 x 50
848

Children should be expected to maintain this systematic approach to multiplying numbers, working right to left along the bottom number. This will ensure that mistakes are not made by 'missing' parts, especially when multiplying numbers with more digits.
</td>
<td></td>
</tr>
<tr>
<td>

Long multiplication

- Each digit continues to be multiplied by each digit, but the totals are recorded in a more compact form, using 'carrying'
- Children's understanding of place value is vital so they recognise when they are multiplying tens, hundreds etc. they record their answer in the correct columns.
- Children should be able to explain each step of the process, initially relating it back

</td>
<td>
124 x 26 becomes
1 2
1 2 4
× 2 6
7 4 4
2 4 8 0
3 2 2 4
1 1
Answer: 3224

6 x 4 = 24 so record the 4 in the units and carry the 20 (2) into the tens 6 x 20 = 120 + (the carried) 20 = 140 so record the 40 in the tens and carry the 100 (1) into the hundreds column. 6 x 100 = 600 + (the carried) 100 = 700. Record as 7 in the hundreds.

20 x 4 = 80 so record this on a new answer row in the correct columns. 20 x 20 = 400. Record the 4 in the hundreds column. 20 x 100 = 2000 so record this appropriately.

Use column addition to add the two totals together, resulting in 3224.
</td>
<td>


Method agreed at staff meeting 22.4.15
</td>
</tr>
</table>

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to previous methods and experiences. They should be able to articulate the different stages of this calculation with the true values of the digits they are dealing with.	'Carrying' can be done above or below the number, but should be consistent as before to avoid mistakes.	

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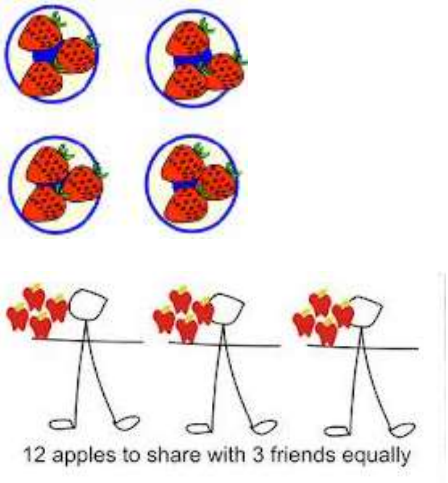
DIVISION

Stage 1: Recording and developing mental images • Children are encouraged, through practical experiences, to develop physical and mental images. • They make recordings of their work as they solve problems where they want to make equal groups of items or sharing objects out equally.	 	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process. Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.
Sharing and Grouping • They solve sharing problems by using a 'one for you, one for me' strategy until all of the items have been given out. • Children should find the answer by counting how many eggs **1 basket** has got. • They solve grouping problems by creating groups of the given	15 eggs are shared between 5 baskets. How many in each basket? First egg to the first basket, 2nd egg to the second etc	

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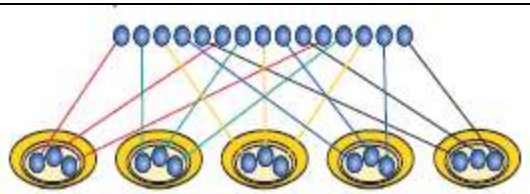
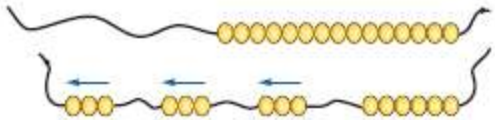
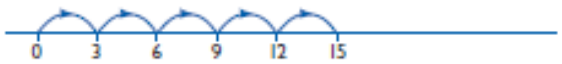
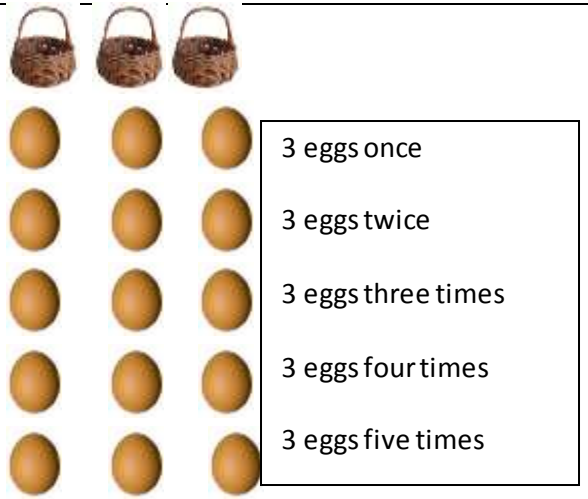
<table>
<tr>
<td>number.
• Children should find the answer by counting out the eggs and finding out how many groups of 3 there are.
• They will begin to use their own jottings to record division</td>
<td> 
There are 15 eggs. How many baskets can we make with 3 eggs in?
  </td>
<td></td>
</tr>
<tr>
<td>Stage 2: Bead strings, number lines simple multiples
• Using a bead string, children can represent division problems
• They count on in equal steps based on adding multiples up to the number to be divided.
• When packing eggs into baskets of three they count in threes - grouping
• If the problem requires 15 eggs to be shared between 3 baskets, the multiple of three is obtained</td>
<td>15 eggs are placed in baskets, with 3 in each basket. How many baskets are needed?
 
Counting on a labelled and then blank number lines.
15 ÷ 3 = 5
 
</td>
<td> 
</td>
</tr>
</table>

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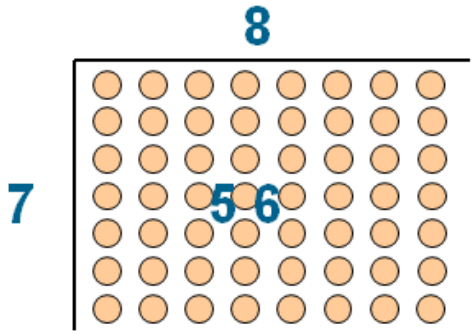
each time all three baskets have received an egg.		
• **Stage 3: Arrays for division** Children construct arrays by grouping the dividend into groups of the divisor. The number of groups made is recorded as the quotient.	The use of arrays help to reinforce the link between multiplication and division  Divided (56) ÷ divisor (7) = Quotient (8)	

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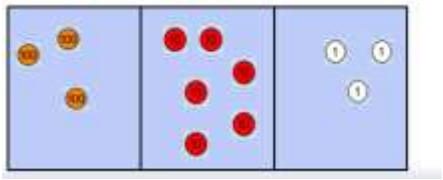
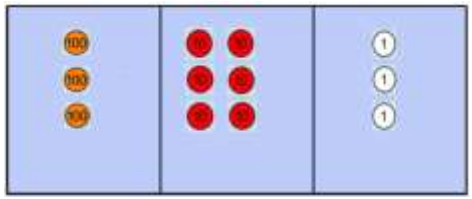
Children then begin to construct the arrays using place value equipment to represent the dividend.	363 ÷ 3  Using the principles of arrays linked to place value becomes:  Each part of the number is grouped or shared into the divisor. Explaining the recording of the division as 1 2 1 3 | 3 6 3	This can then be explained in two ways In one of the three groups, there is one hundred, two tens and one one, making one hundred and twenty one OR There is 1 group of three hundreds, 2 groups of three tens and 1 group of three ones making one hundred and twenty one

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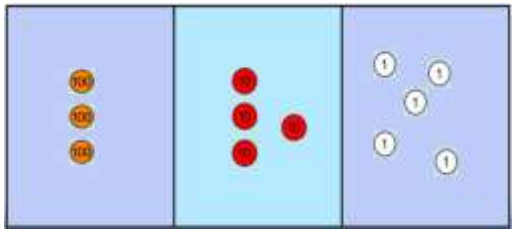
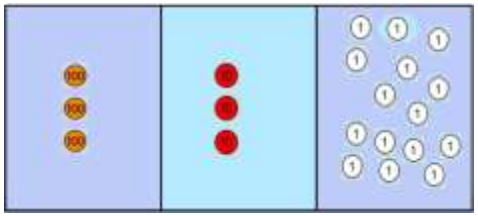
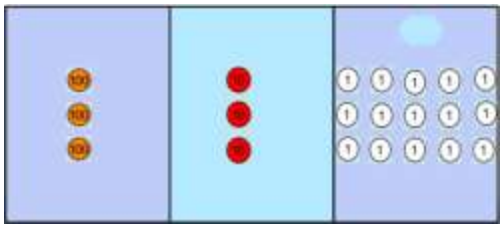
	This then becomes more complex when exchange is needed as complete groups of the divisor cannot be made eg.  Then becomes  Which finally becomes  Recorded as	

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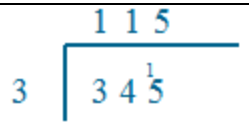
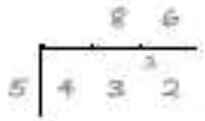
		
Stage 5: Short and Long division Once children have developed a sound understanding of division, using the manipulatives 'formal written methods' of short and then long division. For calculations where numbers with up to 4 digits are divided by a single digit number, children are expected to use short division.	**Stage 5** Short division 432 ÷ 5 becomes  Answer: 86 remainder 2 With short division, children are expected to 'internalise' the working from above	 Agreed at staff meeting 22.4.15

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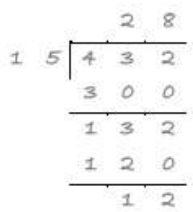
For calculations where numbers of up to 4 digits are divided by a two digit number, children are expected to use long division.	Long division 432 ÷ 15 becomes  Answer: 28 remainder 12 . Children may choose to record the 'chunks' alongside to help them calculate the final answer	By the time children are ready for long division, manipulatives may not aid calculating, however they may aid the understanding of the process of long division. The steps followed can be described as those followed when using PVCs to divide e.g. How many groups of 15 hundreds can we make? None so we exchange the 4 hundreds for 40 tens. How many groups of 15 tens can we make? 2, equivalent to 300. We record the 2 and subtract the 300 that we have 'organised' from the dividend. We are now left with 132 'ones'. How many groups of 15 can we make with these? 8 and we have 12 left over.

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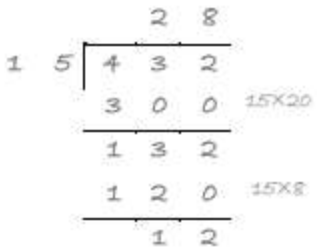
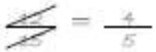
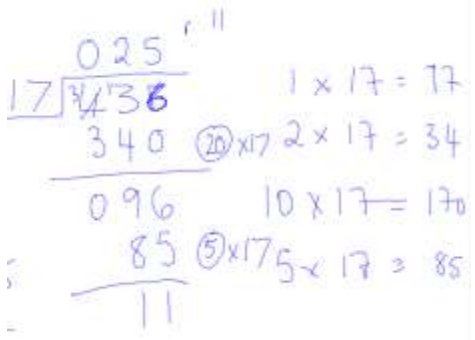
	432 ÷ 15 becomes   Answer: 28 $^{4}/_{5}$ And will start to interpret the 'remainder' in the most appropriate way to the context of the question.	 Agreed at staff meeting 22.4.15

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